Online Auctions: There can be only one.

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***Abstract*—In recent years, the proliferation of the world wide web has lead to an increase in the number of public auctions on the internet. One of the characteristics of online auctions is that a successful implementation requires a high volume of buyers and sellers at its website. Consequently, auction sites which have a high volume of traffic have an advantage over those in which the volume is limited. This results in even greater polarization of buyers and sellers towards a particular site. This is often referred to as the "network effect" in a variety of web and telecommunication applications involving interactions among a large number of entities. While this effect has qualitatively been known to increase the value of the overall network, its effect has never been modeled or studied rigorously. In this paper, we construct a Markov Model to analyze the network effect in the case of web auctions. We show that the network effect is very powerful for the case of web auctions and can result in a situation in which one auction can quickly overwhelm its competing sites. This results in a situation in which the natural stable equilibrium is that of a single online auction seller for a given product and geographical locality. While a single player structure is unlikely because of some approximation assumptions in the model, the trend seems to show the likely existence of single dominant player in the web auction space.**

***Keywords*-Online Auctions, Network Effect**

I. Introduction

A popular method for constructing electronic commerce transactions on the web is that of *online auctions*. A number of sites such as *Ebay*, and *Priceline.com* [16], [17] routinely conduct auctions on the web in order to match buyers and sellers over a variety of products. The process of conducting auctions in the web space has emerged as a credible alternative to that of traditional retailing because of the efficiency of the web in matching potential buyers and sellers. Consequently, the online auction problem has received considerable attention in recent years [2], [3], [4], [6], [7], [8], [9], [10], [11], [13].

The auction space provides special opportunities to both buyers and sellers for the following reasons:

• For sellers, the auction space provides an opportunity to sell products in a fast and time-controlled way. This phenomenon can be clearly be seen in both *Ebay* and *Priceline.com* in which a large fraction (or all in the case of Priceline.com) of the products are liquidations of unsold inventory. The auction site needs to have a large number of buyers in order for such a strategy to be successful.

• For buyers, the relatively low prices of an auction provide great attraction at the expense of some flexibility. This flexibility is usually exhibited in terms of temporal and physical characteristics of buying the items.

The above observations suggest that the value of an intermediary auction system greatly increases with the number of users. This is known as the *network effect*. This effect should not be confused with economies of scale in traditional systems which level off at a certain point. In contrast, the network effect of the auction system continues to grow in a self-sustaining way with site popularity. Such a network effect is also present in any system such as *community-based* search engines [18] in which the quality of the results can depend upon the number of users utilizing the system. We note that the exact value, sustainability and impact of the network effect deeply depends upon how it is leveraged for a particular application.

We note that our analysis pertains to the case of auction sites which act as an *intermediary* to match potential buyers and sellers. For example, in the case of *Ebay*, the product is sold by an independent seller who is charged a fee for the transaction. While some online auctioneers (such a Policeauctions.com) directly auction their own items, this is not the model analyzed in this paper. This is because the latter model does not play a *matching role* between many buyers and sellers, which is critical for the network effect. Correspondingly, such sites also have less capacity in attracting diverse buyers to their site, or are limited to buyers interested in a particular segment of the fragmented marketplace. *In this paper, we are directly looking only at auction sites in which the aim of the auctioneer is to act purely as an intermediary between the buyers and sellers.* Our analysis applies to the case where sellers and buyers act as independent agents in the auctioning process, either of whom can be charged a fee by the auctioneer. Thus, the auctioning process is purely a *intermediary service oriented* business provided to the buyer and seller.

This paper will analyze the network effect [12] for online auctions. The results will show that the network effect provides an overwhelming advantage to the dominant auctioneer within a given product and geographical space. This advantage seems to be sufficient for one auctioneer to quickly gain market share from other less known auctioneers. While this is unlikely to lead to a purely monopolistic situation because of some approximations in the model, the results show that the advantage enjoyed by a single service-oriented dominant player (such as Ebay) can be quite powerful.

This paper is organized as follows. In the next section, we will discuss the network effect for online auctions. We will construct a Markov Model which determines the equilibrium stability for online auctions. In section III, we will examine some characteristics of this model, the corresponding interpretation, and the equilibrium stability. In section IV, we will provide experimental simulation and discussions.

II. The Auction Model

In this section, we will model the network effect in online auctions. In order to do so, we will construct a Markov Model which relates the behavior of buyers and sellers in an online environment. The basic assumptions in the model are as follows:

- The auctioneer acts only as an intermediary agent during the selling process, and is not a direct party to the transaction. Therefore, buyers and sellers are only interested in the most efficient and cost effective transaction.
- A greater number of buyers provides a more effective and cost efficient transaction to the seller. Therefore, a seller is more likely to choose a particular auctioning agent, if it provides access to a greater number of buyers.
- A greater number of sellers provides a more effective and cost efficient transaction to the buyer. Therefore, an auction with a larger number of sellers is also more attractive to the buyer.
- The buyers and sellers should be able to effectively assess the number of buyers and sellers at different auction sites. This is uniquely true of online auctions, where the number of sellers, number of bids on different items, item-specific traffic, and final bid price over different times are quite transparent. Furthermore, since online auctions for each item run over the course of several days at different sites, it is possible to compare the effectiveness of different auctioneers easily.
- Each auction site must have its own set of buyers and sellers and is not able to match with bids made at other auction sites. We note that brokerages are also a form of online auctions, but since they match buyers and sellers across different brokerages, they do not satisfy this assumption.

We note that the above observations serve as the source of the network effect in online auctions. It is clear that auctions with a larger number of buyers are likely to attract more sellers and vice-versa. This may result in *defection behavior* from one auction to another. This defection behavior creates a self-sustaining effect which results in one dominant auction crowding out the others rapidly. Even in situations in which multiple auctions exist with an equal amount of market dominance, we will see that the nature of the equilibrium is stochastically unstable, and will lead to dominance of one auction over time.

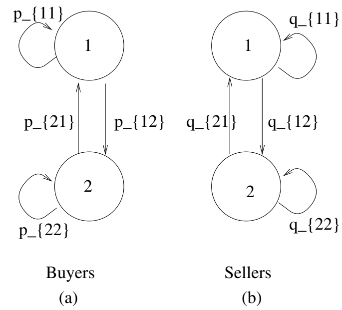

Figure 1. Static Markov Models for Buyer and Seller Defection

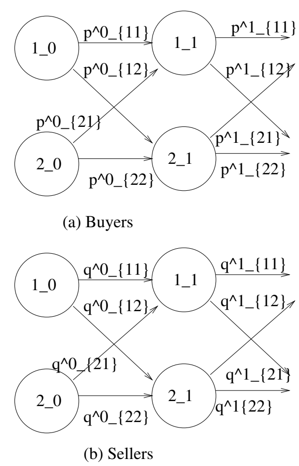

Figure 2. Dynamic Markov Models for Buyer and Seller Defection

In order to formally model the propensity of buyers and sellers to pick the most attractive auction, let us consider two competing auctions, each of which has a set of buyers and sellers. We also assume that these are the only two available auctions, and a hypothetical buyer must choose between one of the two. Such a binary model can be directly generalized to the case of multiple auctions using the binary model recursively. In Figure 1, we have illustrated the defection model in which we have illustrated two hypothetical auctioneers. Two separate markov models are illustrated. The model on the left (Figure 1(a)) illustrates the defection behavior of a hypothetical buyer. Thus, the hypothetical buyer may shift between states 1 and 2. These states correspond to his choosing auctions 1 or 2 respectively. Thus, an auction buyer may use auction 1 for a current transaction, but may transition to auction 2 for the next transaction with probability p_{12}. Alternatively, they may choose to use the same auction with probability p_{11}. The latter case is illustrated as a self loop in state 1 of Figure 1(a). It is also clear that since the buyer uses either auction 1 or auction 2 for each transaction, we have:

$$p_{11} + p_{12} = 1 \quad (1)$$

$$p_{21} + p_{22} = 1 \quad (2)$$

In general, we would like to find the steady-state probability that the buyer chooses one of the two auctions. We assume that the steady state probability of states 1 and 2 are denoted by α_1 and α_2 respectively. Then, for the Markov model to be in steady state, the transition probability into a given state must be equal to the transition probability out of it. For state 1, we have the following relationship:

$$\alpha_1 \cdot p_{11} + \alpha_1 \cdot p_{12} = \alpha_1 \cdot p_{11} + \alpha_2 \cdot p_{21} \quad (3)$$

This relationship is equivalent to the following:

$$\alpha_1 \cdot p_{12} = \alpha_2 \cdot p_{21} \quad (4)$$

We can set up a similar steady state relationship for state 2, though the relationship is equivalent to that of Equation 4. We also note that the sum of the steady state probabilities over the buyer (or seller) side Markov Model must be equal to 1. Therefore, we have:

$$\alpha_1 + \alpha_2 = 1 \quad (5)$$

We note that we can set up similar relationships at the seller side as well. Let us assume that the steady state probability for states 1 and 2 at the seller side are denoted by β_1 and β_2 respectively. Correspondingly, we have the following relationships among the transition and steady state probabilities:

$$q_{11} + q_{12} = 1 \quad (6)$$

$$q_{21} + q_{22} = 1 \quad (7)$$

$$\beta_1 \cdot q_{12} = \beta_2 \cdot q_{21} \quad (8)$$

So far, we have not connected the state probabilities of the Markov Models corresponding to Figures 1(a) and (b). We earlier observed that the likelihood of the defection of a buyer is dependent upon the number of sellers present at an auction and vice-versa. This effectively means that the transition probabilities of the buyer-model are dependent on the state probabilities of the seller-model and vice-versa. This is a unique way of relating two markov models, since the transition probabilities of one model depend upon the state probabilities of the other and vice-versa. This relationship is defined by the following two non-increasing functions $f(\cdot)$ and $g(\cdot)$:

$$p_{12} = f(\beta_1), p_{21} = f(\beta_2)$$

$$q_{12} = g(\alpha_1), q_{21} = g(\alpha_2)$$

We refer to these functions as *defection functions*. The function $f(\cdot)$ is the *buyer defection function*, and the function $g(\cdot)$ is the *seller defection function*. The functions $f(\cdot)$ and $g(\cdot)$ should satisfy the following properties:

- $f(\cdot)$ and $g(\cdot)$ are both non-increasing functions. This corresponds to the fact that a higher state probability at the buyer side of an auction corresponds to a lower probability of seller defection from that auction and vice-versa.
- The functions are defined over the probability range $(0, 1)$ and satisfy the following end-point relationships:

  $$f(0) = 1, f(1) = 0, g(0) = 1, g(1) = 0$$

  These constraints correspond to the fact that a buyer will not participate in any auction which has no sellers and vice-versa.
- The above-mentioned principle can be generalized further to use a *minimum critical mass* in order to define a threshold at which the buyers or sellers will not participate. This minimum critical mass is required for an auction to be a viable activity. We denote this minimum critical mass for the buyers and sellers by c_b and c_s respectively. Below this critical mass, a buyer or seller in the auction has 100% defection probability.

First, we will begin by defining a simple linear functional form for relating the probability of defection to the steady state probabilities. This linear function $f(x)$ is defined in the probability range $(0, 1) \Rightarrow (0, 1)$ as follows:

$$f(x) = \begin{cases} 1 & 0 \le x \le c_s \\ (1 - c_s - x)/(1 - 2 \cdot c_s) & c_s \le x \le 1 - c_s \\ 0 & 1 - c_s \le x \le 1 \end{cases}$$

The corresponding function $g(x)$ is defined similarly except that the critical mass c_b for buyers is utilized. Therefore, we have:

$$g(x) = \begin{cases} 1 & 0 \le x \le c_b \\ (1 - c_b - x)/(1 - 2 \cdot c) & c_b \le x \le 1 - c_b \\ 0 & 1 - c_b \le x \le 1 \end{cases}$$

We note that the above simple function is a natural choice based on the constraints discussed earlier. We will analyze the behavior of the Markov model using this function. We make the following observations:

Observation 2.1: A steady state solution to the markov model is $\alpha_1 = 1$, $\alpha_2 = 0$, $\beta_1 = 1$, and $\beta_2 = 0$.

We note that because of the values of α_1 and β_1, the values of the transition probabilities are as follows:

$$p_{11} = 1, p_{12} = 0, p_{21} = 1, p_{22} = 0$$

By substituting these values, we can satisfy all the conditions discussed above. Similarly, we can show the following results:

Observation 2.2: A steady state solution to the markov model is $\alpha_1 = 0$, $\alpha_2 = 1$, $\beta_1 = 0$, and $\beta_2 = 1$.

Observation 2.3: A steady state solution to the markov model is $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\beta_1 = 0.5$, and $\beta_2 = 0.5$.

We note that for general functions $f(\cdot)$ and $g(\cdot)$, the following relationships hold true:

Lemma 2.1: A steady state solution to the problem can be defined if and only if a pair $(\alpha_0, \beta_0) \in (0, 1)$ can be found for which the buyer and seller defection functions satisfy the following relationship:

$$\alpha_0 \cdot f(\beta_0) = (1 - \alpha_0) \cdot f(1 - \beta_0) \tag{9}$$

$$\beta_0 \cdot g(\alpha_0) = (1 - \beta_0) \cdot g(1 - \alpha_0) \tag{10}$$

Proof: First, let us consider the case when such a pair (α_0, β_0) can be found satisfying the conditions of Equations 9 and 10. A steady state solution can be determined by setting the parameters as follows:

$$\begin{aligned} \alpha_1 = \alpha_0, \alpha_2 = 1 - \alpha_0 \\ \beta_1 = \beta_0, \beta_2 = 1 - \beta_0 \\ p_{21} = f(1 - \beta_0), p_{12} = f(\beta_0) \\ p_{11} = 1 - p_{12}, p_{22} = 1 - p_{21} \\ q_{21} = g(1 - \alpha_0), q_{12} = g(\alpha_0) \\ q_{11} = 1 - q_{12}, q_{22} = 1 - q_{21} \end{aligned}$$

It is easy to verify that all the steady state conditions for the markov model are satisfied by these conditions. For example, the conditions illustrated in Equations 4 and 8 are satisfied because of the pre-conditions in the problem statement of this lemma. The conditions interrelating the two models are satisfied because of the way in which the transition probabilities are chosen.

Next, if we consider a Markov Model in steady state, then the values of (α_0, β_0) can be chosen as the (respective) buyer and seller state probabilities of one of the two auctions. From the steady state conditions, it can be shown that the pre-conditions of the lemma are satisfied. ■

We note that one of the consequences of this lemma is that it provides conditions on the existence of a steady state in the markov chain. *Furthermore, any steady state can be satisfied by the pair (α_0, β_0) which satisfy the above conditions.* These conditions can also be used to show the following:

Corollary 2.1: For linear defection functions with non-zero critical mass, the only steady state pairs are defined by $(\alpha_0, \beta_0) = (0, 0)$, $(\alpha_0, \beta_0) = (1, 1)$, and $(\alpha_0, \beta_0) = (0.5, 0.5)$.

Proof Sketch: It is straightforward to verify our earlier observations that the specified pairs $(0, 0)$, $(1, 1)$, and $(0.5, 0.5)$ are steady state conditions. In order to prove the reverse, let us pick values (α_0, β_0) of the probability which are not equal to $(0, 0)$, $(0.5, 0.5)$, and $(1, 1)$ respectively. We will show that it is not possible to pick values of (α_0, β_0) which are such that Equations 9 and 10 are satisfied. Different cases can be considered for the ranges of α_0 and β_0 with respect to the corresponding critical mass. For each of these cases, it can be shown that the Equations 9 and 10 cannot be consistently satisfied by such a pair.

III. Stable and Unstable Equilibrium

We note that not all steady state conditions are equivalent in terms of stability. Intuitively, a steady state condition is stable when a small disturbance to the probability of that state results in the markov model returning to the earlier state of equilibrium. We note that the issue of stability arises in our application because of the special relationship between two Markov Models: the state probabilities of one depend upon the transition probabilities of the other, and vice-versa. In the standard Markov Model, all steady states are stable and vice-versa. In order to define the concept of stability of the steady state in markov models more exactly, we need to define a *dynamic version* of the markov model. In the dynamic version of a markov model, we define *temporally layered states*, in which the ith layer corresponds to the ith transition. Temporally layered markov models are a useful technique for understanding the transient behavior of markov models, and the rate at which a given markov model will reach equilibrium.

For each state i of the standard markov model, the temporally layered markov model has a state i_T for the state i after T transitions. The state i_0 corresponds to the initial probability of state i. For each edge (i, j) in the initial Markov Model, we have an edge (i_t, j_{t+1}) in the transformed Markov Model. The value of t can range from 0 to ∞. An example of the dynamic Markov Model is illustrated in Figure 2. The state probability *at time period* t on the buyer side for nodes i_t^1 and i_t^2 are denoted by α_1^t and α_2^t respectively. We note that the state probabilities of the node layer t at all time periods other than at time period t, are equal to zero. This is because of the layered structure of the Markov Model in which a transition occurs into layer t only after t time periods. The corresponding state probabilities on the seller side are denoted by β_1^t and β_2^t respectively. The transition probability of the edge (i_t, j_{t+1}) on the buyer side is denoted by p_{ij}^t. The corresponding transition probability for the seller side of the Markov Model is denoted by q_{ij}^t. As in the static Markov Model, the transition probabilities are related to the state probabilities. In this case, the transition probabilities at time period t at the buyer side are related to the state probabilities at time period t and vice-versa.

$$\begin{aligned} p_{12}^t = f(\beta_1^t), p_{21}^t = f(\beta_2^t) \\ q_{12}^t = g(\alpha_1^t), q_{21}^t = g(\alpha_2^t) \end{aligned}$$

The corresponding transition equations in the Markov Model are defined as follows:

$$\alpha_1^t \cdot p_{11}^t + \alpha_2^t \cdot p_{21}^t = \alpha_1^{t+1} \cdot (p_{11}^{t+1} + p_{12}^{t+1}) \tag{11}$$

Since the sum of the transition probabilities out of a state is 1:

$$\alpha_1^t \cdot p_{11}^t + \alpha_2^t \cdot p_{21}^t = \alpha_1^{t+1} \tag{12}$$

The corresponding transition condition on the seller side is defined as follows:

$$\beta_1^t \cdot p_{11}^t + \beta_2^t \cdot p_{21}^t = \beta_1^{t+1} \tag{13}$$

In general, as $t \rightarrow \infty$ the state probabilities α_1^t and α_2^t will move to one of the steady states. In the standard form of the Markov Model, only one steady state exists which is independent of the initial state probabilities α_1^0 and α_2^0. However, in the form of the model discussed in this paper, (in which the transition probabilities of one model depend upon the state probabilities of the other), the final state probabilities depend both upon the initial state probabilities as well as the initial state of the Markov Model. In addition, the Markov Model is more likely to rest in a final steady state which is *stable*. Consequently, we will define the concept of stability of steady state. In essence the steady state condition states that a small perturbation from the state distribution is likely to bring it back to its original state.

Definition 3.1: A steady state (α, β) is said to be stable, if for any small perturbation vector ϵ such that $|\epsilon| \leq \epsilon_0$, a starting state of $(\alpha_0, \beta_0) = (\alpha, \beta) + \bar{\epsilon}$ leads to (α, β) as the final state. Therefore, we would have:

$$\lim_{t \Rightarrow \infty} (\alpha_t, \beta_t) = (\alpha, \beta). \tag{14}$$

The intuitive significance of the above definition is that a small perturbation from the steady state is likely to lead to the system reverting back to its steady state. The aim of this is to model real life situations in which minor transitory events lead to the system being perturbed from its steady state. In such cases, stable steady states are more likely to reflect the final behavior of the model. We make the following conjectures about the behavior of these models:

Conjecture 3.1: The solution $(\alpha, \beta) = (0.5, 0.5)$ is not stable.

The intuition behind this conjecture is that a reduction of the state probabilities below 0.5 for a particular auction on the buyer side reduces the state probability at the seller side as well. (This is because of the functional relationship between the state and transition probabilities.) the corresponding transition probabilities on the seller side and vice-versa. This leads to a self sustaining cycle of reduction on state probabilities for one of the two auctions. We currently do not have a formal proof of this behavior, but will illustrate the process through simulation. Other observations about the state probabilities are as follows:

Observation 3.1: The solutions $(\alpha, \beta) = (0, 0)$ and $(\alpha, \beta) = (1, 1)$ are stable.

This observation is straightforward, because any perturbation which is lower than the critical mass reverts the system to the steady state in the very next iteration.

The implications of the above observations are that the auction system is in a *stable* steady state in case of one or the other auction dominating. In the next section, we will provide some empirical simulations which illustrate the extent of the domination of the states.

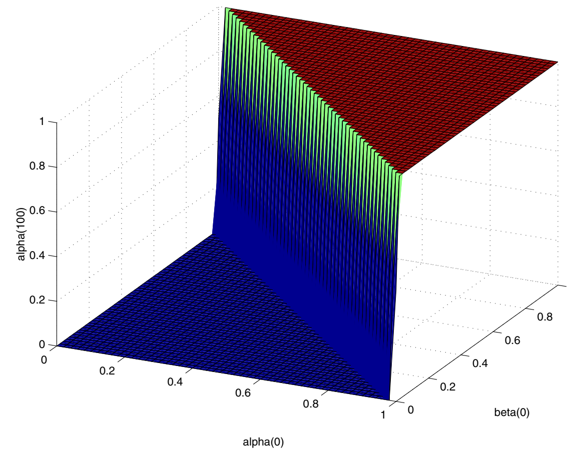

Figure 3. Final State of Model with Different Starting States

A. Adding Temporal Memory to the Model

We note that the transition probabilities on the buyer side during a given time period are dependent upon the state probabilities on the buyer side and vice-versa. In the model discussed above, we have assumed that the transition probabilities during time period t are dependent upon the state probabilities during the same time period t. This is purely a *memoryless* model, since the state probabilities during earlier periods do not affect the transition probability. In practice, there may be a delay in the process of buyers or sellers reacting to state changes in the Markov Model. Therefore, we define a *time averaged* state probability as follows:

$$\alpha_1'^t = \frac{\sum_{i=i}^{t} \alpha_1^i \cdot 2^{i-t\cdot\lambda}}{2^{i-t\cdot\lambda}} \tag{15}$$

The value of $1/\lambda$ is defined as the half-life during which the importance of a state probability gets reduced by half. We note that for very large values of λ, this corresponds to the purely memory less model. The transition probabilities during the time period t are defined as a function of the time-averaged state probabilities during the same time period.

$$p_{12}^t = f(\beta_1'^t), p_{21}^t = f(\beta_2'^t)$$
$$q_{12}^t = g(\alpha_1'^t), q_{21}^t = g(\alpha_2'^t)$$

We note that the addition of memory to the model generally affects the rate of convergence to the model, but it does not usually affect the state to which the system converges. A lower value of λ increases the half life for calculating the relationship between the transition probabilities and the state probability of the different Markov Models. This also increases the time required by the model to reach steady state.

IV. Experimental Simulations and Discussion

In this section, we will illustrate some experimental simulations which confirm the convergence behavior of the markov model for online auctions. An important issue to be examined is how the initial choice of state probabilities affects the final state probabilities in the model. In Figure 3, we have illustrated the behavior of the final state probabilities (after 100 transitions of the Markov chain) with different initial state probabilities. In this case, the parameters were set at $c_b = c_s = 0.1$ and $\lambda = 1$. The value of (α, β) was allowed to vary over the entire range of possibilities in the unit square over $50 * 50 = 2500$ points in the grid. It is interesting to notice that over almost the entire range of values, the model converged to either (0,0) or (1, 1). For example, even when using the value of $\alpha_0 = 0.5$, $\beta_0 = 0.48$, the system converged to the value $(0, 0)$. Intuitively, this means that even if one of the two auctions had a slight advantage over *either* the buyer or seller side, this advantage is sufficient for one auction to overwhelm the other. This is evidence of the fact that the state $(0.5, 0.5)$ does not correspond to a stable steady state. The only set of initial states (α_0, β_0) for which the system did not always converge to either $(0, 0)$ or $(1, 1)$ was the set of values in the grid along the line $\alpha_0 + \beta_0 = 1$. As is evident from Figure 3, for the entire range of values along each side of this dividing line, the final state of the model takes on the value $(0, 0)$ or $(1, 1)$. The only set of values which converged to $(0.5, 0.5)$ were found along the dividing line. We also note that in some cases, small rounding errors lead to the system converging to $(0, 0)$ or $(1, 1)$, whereas the final state ought to be $(0.5, 0.5)$. This is again an evidence of just how unstable the state $(0.5, 0.5)$ is in practice, since such errors are also likely to be manifested as probabilistic variations which are inherent in real life. In many cases, 100 transitions were not sufficient to reach convergence at this point. This brings to the natural observation that a greater level of skew in the initial starting states required a greater level of iterations for convergence. More detailed experiments on the rate of convergence and the dynamic variation of the model may be found in [1].

Thus, the results of this paper provide an understanding of the network effect in online auctions. The presence of the network effect is a natural consequence of the fact that auctions need a critical mass to operate, and the stable equilibrium results in one auction reducing others to below critical mass. We also note that the results of our paper are primarily applicable to *intermediary service-oriented auctions*, because the model requires independently acting buyers and sellers for the presence of the network effect. Further, we note that the presence of software agents may allow concurrent bidding, though this is only a small fraction of the overall auction process, and not likely to affect the model. We conjecture that some of these results are likely to partially apply to other network effect driven systems such as search engines calculating page rank and importance with user browsing behavior [18].

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