Recommendations in Signed Social Networks

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ABSTRACT

Recommender systems play a crucial role in mitigating the information overload problem in social media by suggesting relevant information to users. The popularity of pervasively available social activities for social media users has encouraged a large body of literature on exploiting social networks for recommendation. The vast majority of these systems focus on unsigned social networks (or social networks with only positive links), while little work exists for signed social networks (or social networks with positive and negative links). The availability of negative links in signed social networks presents both challenges and opportunities in the recommendation process. We provide a principled and mathematical approach to exploit signed social networks for recommendation, and propose a model, RecSSN, to leverage positive and negative links in signed social networks. Empirical results on real-world datasets demonstrate the effectiveness of the proposed framework. We also perform further experiments to explicitly understand the effect of signed networks in RecSSN.

General Terms

Algorithms

Keywords

Social Recommendation; Signed Networks; Negative Links

1. INTRODUCTION

Recommender systems play a crucial role in helping online users collect relevant information by suggesting information of potential interest. The increasing popularity of social media allows online users to participate in online activities in a pervasive way. These social networks provide independent sources of recommendation and unleash previously unknown potentials of recommender systems. The exploitation of social networks for recommendation has attracted increasing interest in recent years [3, 16, 6, 17, 7]. Existing social recommender systems can be roughly categorized into memory-based systems and model-based systems [32]. The vast majority of these systems focus on unsigned social networks (or social networks with only positive links); however, social networks in social media can contain both positive and negative links. Examples of these signed social networks include Epinions1 with trust and distrust links, and Slashdot2 with friend and foe links. Such networks provide a much richer source of information than what is exploited by the current systems [39, 13, 1, 5].

Experience with real-world social systems such as Epinions and eBay suggests that negative links in signed social networks are at least as important as positive links [4]. Negative links tend to be more noticeable and credible, and weighed more than positive links of a similar magnitude [21, 2]; therefore, they can be critical in various analytical tasks. For example, negative links add a significant amount of knowledge than that embedded only in positive links [12, 29], and a small number of negative links can improve the performance of positive link prediction remarkably [4, 14]. Evidence from recent achievements in signed social network analysis suggests that negative links may also be potentially helpful in recommender systems. However, negative links exhibit very different properties from positive links [28, 33]; hence, recommendation with signed social networks cannot be successfully carried out by simply extending recommender systems with unsigned social networks in a straightforward way. For example, most existing recommender systems with unsigned social networks assume that a user’s preference is similar to or influenced by their friends (or positive links) according to homophily [22] and social influence [19]. Such assumptions are not applicable in signed social networks [33]. This makes the recommendation problem more challenging in the signed network scenario.

In this paper, we study the problem of recommendation with signed social networks, in the context of (i) exploiting positive and negative links in signed social networks; and (ii) modeling them mathematically for recommendation. In order to address these two challenges, we propose the RecSSN framework, in which the primary contributions are as follows:

- We provide a principled approach to mathematically exploit signed social networks for recommendation;

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• We propose a novel recommendation framework, denoted by RecSSN, which captures both positive and negative links in signed social networks; and

• We evaluate the proposed framework in real-world social media datasets to understand the effectiveness and mechanisms of the proposed framework.

The remainder of this paper is organized as follows. In Section 2, we formally define the problem of recommendation with signed social networks. We describe the datasets and perform preliminary data analysis on these datasets in Section 3. In Section 4, we provide approaches to model signed social networks and introduce details about the proposed RecSSN framework with an optimization algorithm. Section 5 presents experimental results with discussions. Section 6 briefly reviews related work. Finally, Section 7 concludes with future work.

2. PROBLEM STATEMENT

Let \( U = \{u_1, u_2, \ldots, u_N\} \) be the set of users and \( V = \{v_1, v_2, \ldots, v_m\} \) be the set of items where \( N \) and \( m \) are the numbers of users and items, respectively. We assume that \( R \in \mathbb{R}^{N \times m} \) is the user-item matrix where \( R_{ij} \) denotes an observed score from \( u_i \) to \( v_j \) and we set \( R_{ij} = 0 \) if the score from \( u_i \) to \( v_j \) is missing. Note that in different recommender systems, the score has different meanings. For example, in rating systems such as Epinions and Netflix, scores denote rating scores from users to items; in tagging systems such as Slashdot and Flikcr, scores indicate whether users are associated with tags.

For the problem of recommendation with signed social networks, signed social networks among users are also available in addition to the user-item matrix \( R \). A signed social network \( G \) can be decomposed into a positive component \( G^p \) and a negative component \( G^n \). Let \( A^p \in \mathbb{R}^{N \times N} \) be the adjacency matrix of \( G^p \) where \( A^p_{ij} = 1 \) if \( u_i \) and \( u_j \) are positively linked and \( A^p_{ij} = 0 \) otherwise. Similarly, \( A^n \in \mathbb{R}^{N \times N} \) denotes the adjacency matrix of \( G^n \) where \( A^n_{ij} = 1 \) if \( u_i \) and \( u_j \) are negatively linked and \( A^n_{ij} = 0 \) otherwise. Note that we only consider links with a binary weight \( \{0, 1\} \) in this paper although the generalization of the proposed framework to links with continuous weights is straightforward.

With the aforementioned notations and definitions, the problem of recommendation with a signed social network can be formally stated as follows:

**Given observed values in \( R \) and a signed social network \( G \) with positive links \( A^p \) and negative links \( A^n \), the problem of recommendation with a signed social network aims to infer missing values in \( R \).**

3. DATA ANALYSIS

Because recommendation with unsigned networks strongly depends on the finding that users are likely to share similar preferences with their friends [32], it is natural to explore similar findings of signed social networks for recommendation. Such an understanding lays the groundwork for a meaningful recommendation framework with signed social networks. In this section, we first introduce the datasets and then perform preliminary data analysis to understand the impact of signed social networks on recommendation.

| Table 1: Statistics of the Epinions and Slashdot datasets. |
|-----------------|----------------|
|                 | Epinions   | Slashdot |
| # of Users      | 18,210     | 11,868   |
| # of Items      | 41,089     | 27,942   |
| # of Positive Links | 358,985  | 290,719  |
| # of Negative Links | 75,091   | 67,108   |
| Density of User-item Matrix | 8.42e-4  | 1.20e-3  |
| # of Users with Negative Links | 11,598   | 7,837    |

3.1 Datasets

For the purpose of this study, we collected two datasets from Epinions and Slashdot. Some details about these datasets are described below.

Epinions is a popular product review site. Users in Epinions can create both positive (trust) and negative (distrust) links to other users, which results in a signed network \( G \). They can also rate various products with scores ranging from 1 to 5. Therefore, if \( u_i \) rates \( v_j \), \( R_{ij} \) is the rating score, and \( R_{ij} = 0 \) otherwise.

Slashdot is a technology news platform. Users in Slashdot can create friend (positive) and foe (negative) links to other users, which results in the signed network \( G \). They also can specify tags associated with them. Therefore if \( v_j \) is associated with \( u_i \), \( R_{ij} = 1 \), and \( R_{ij} = 0 \) otherwise.

Some additional preprocessing was performed on these two datasets by filtering users without any positive and negative links, or with few non-zero entities in the user-item matrix \( R \). A number of key statistics of these datasets are illustrated in Table 1.

It is evident from these statistics that (i) positive links are denser than negative links in signed social networks; (ii) not all users in signed social networks have negative links; and (iii) the user-item matrix is very sparse.

3.2 An Analysis of Signed Social Networks

Previous studies suggest that users in unsigned social networks are likely to share similar preferences with their friends, which serves as the basis of most existing recommender systems with unsigned social networks [35]. In this subsection, we investigate similar preference properties of users in signed social networks.

Let \( p_i \), \( n_i \), and \( r_i \) denote the number of users with positive, negative, and no links with \( u_i \). We construct three circles for each user \( u_i \) with the same size of \( \min(p_i, n_i, r_i) \). These circles correspond to (i) a friend circle \( FR_i \) including randomly selected users who have positive links with \( u_i \); (ii) a foe circle \( FO_i \) containing randomly selected users who have negative links with \( u_i \); and (iii) a random circle \( RA_i \) including randomly selected users who have no links with \( u_i \). Similar to the study of preference properties of users in unsigned social networks [35], we investigate preference properties of users in signed social networks by investigating similarities between users and their circles. We will use the friend circle as an example to illustrate how we perform these investigations.

Let \( F_{k}^p \) be the set of users from \( FR_i \) from whom we observe scores to the item \( v_k \) as

\[
F_{k}^p = \{ u_j | v_j \in FR_i, R_{jk} \neq 0 \} \tag{1}
\]

We will make these two datasets publicly available via http://jiliang.xyz/signed.html
Then, the average score of \( \mathcal{F}_R \) to the \( k \)-th item \( \mathbf{R}_k^{ip} \) is calculated as follows:

\[
\mathbf{R}_k^{ip} = \begin{cases} 
\sum_{v_j \in \mathcal{I}_i} \mathbf{R}_{ij} \cdot \mathbf{R}_j^{ip} & \text{for } \mathbf{R}_k^{ip} \neq \emptyset, \\
0 & \text{otherwise.}
\end{cases}
\] (2)

With \( \mathbf{R}_k^{ip} \), we can calculate the similarity \( s_i^p \) between \( u_i \) and her friend circle \( \mathcal{F}_{R_i} \). In this work, we investigate three ways of calculating \( s_i^p \) as follows:

- **CI**: We compute \( s_i^p \) as the number of common items scored by both \( u_i \) and his/her friend circle \( \mathcal{F}_{R_i} \) as:

\[
s_i^p = |\mathcal{I}_i|, \quad \mathcal{I}_i = \{ v_j | \mathbf{R}_{ij} \neq 0 \land \mathbf{R}_j^{ip} \neq 0 \} \quad (3)
\]

- **COSINE**: The term \( s_i^p \) is calculated as cos similarity of scores between \( u_i \) and \( \mathcal{F}_{R_i} \) over all items as:

\[
s_i^p = \frac{\sum_{v_j} \mathbf{R}_{ij} \cdot \mathbf{R}_j^{ip}}{\sqrt{\sum_{v_j} \mathbf{R}_{ij}^2} \sqrt{\sum_{v_j} \mathbf{R}_j^{ip}^2}},
\] (4)

- **CI-COSINE**: Different from COSINE, CI-COSINE computes the cosine similarity over common items \( \mathcal{I}_i \) as:

\[
s_i^p = \frac{\sum_{v_j \in \mathcal{I}_i} \mathbf{R}_{ij} \cdot \mathbf{R}_j^{ip}}{\sqrt{\sum_{v_j \in \mathcal{I}_i} \mathbf{R}_{ij}^2} \sqrt{\sum_{v_j \in \mathcal{I}_i} \mathbf{R}_j^{ip}^2}}.
\] (5)

Similarly, we can compute the similarity \( s_i^p \) between \( u_i \) and his/her foe circle \( \mathcal{F}_{O_i} \), and the similarity \( s_i^p \) between \( u_i \) and his/her random circle \( \mathcal{R}_{A_i} \). Let \( s_i^p, s_i^a \), and \( s_i^s \) be the similarity vectors of \( s_i^p \), \( s_i^a \), and \( s_i^s \) over users for these three circles, respectively. The means of \( s_i^p, s_i^a \), and \( s_i^s \) are demonstrated in Table 2. We observe that (i) friend circles have larger means than foe circles; and (ii) among these three circles, friends circles have the largest means. From these two observations, we form two assumptions about social signed networks: (i) users are likely to be similar with their friend circles; and (ii) users are likely to be more similar with their friend circles than their foe circles.

For two vectors \( \{x, y\} \), the null hypothesis \( H_0 \) and the alternative hypothesis \( H_1 \) of a two-sample \( t \)-test are defined as follows:

\[
H_0 : x \leq y \quad H_1 : x > y.
\] (6)

where the null hypothesis indicates that the mean of \( x \) is less than or equal to that of \( y \). We perform \( t \)-test on \( \{s_i^p, s_i^s\} \) and \( \{s_i^p, s_i^s\} \) to examine aforementioned assumptions, respectively. For example, when we perform the \( t \)-test on \( \{s_i^p, s_i^s\} \), the null hypothesis is that users are likely to be less similar with their friend circles than their foe circles; therefore if we reject the null hypothesis, then the assumption of users likely to be more similar with their friend circles than their foe circles is verified. The null hypothesis for each test is rejected at significance level \( \alpha = 0.01 \) with \( p \)-values shown in Table 3. The evidence from \( t \)-test on \( \{s_i^p, s_i^s\} \) suggests that users are likely to be similar with their friend circles; and the evidence from \( t \)-test on \( \{s_i^p, s_i^s\} \) indicates that users are likely to be more similar with their friend circles than their foe circles.

### 4. THE PROPOSED FRAMEWORK

Two types of information from unsigned social networks can be exploited for recommendation, which correspond to local information and global information [34]. Local information reveals the correlations among the user and his/her friends, while global information reveals the reputation of the user in the whole social network. Users in the physical world are likely to ask for suggestions from their local friends while they also tend to seek suggestions from users with high global reputation. This suggests that both local and global information can be exploited in social networks to improve the performance of recommender systems [31]. In the following subsections, we will first provide details about the methods for capturing local and global information in signed social networks, and then introduce the proposed RecSSN framework.

Matrix factorization is chosen as our basic model because it is one of the most popular techniques for building recommender systems [11, 10]. Assume that \( U_i \in \mathbb{R}^K \) is the \( K \)-dimensional preference latent factor of \( u_i \), and \( \mathbf{V}_j \in \mathbb{R}^K \) is the \( K \)-dimensional characteristic latent factor of item \( j \). Typically, scores from \( u_i \) to \( v_j \) in \( \mathbf{R}_{ij} \) are modeled by the interactions between their latent factors. This interaction is defined in terms of the product of the latent vectors:

\[
\mathbf{R}_{ij} = \mathbf{U}_i^\top \mathbf{V}_j
\] (7)

Matrix factorization-based recommender systems solve the following optimization problem:

\[
\min_{\mathbf{U}_1, \ldots, \mathbf{U}_N} \sum_{i=1}^{N} \sum_{j=1}^{m} \mathbf{W}_{ij} ||\mathbf{R}_{ij} - \mathbf{U}_i^\top \mathbf{V}_j||^2_2 + \alpha(||\mathbf{U}||_F + ||\mathbf{V}||_F) \quad (8)
\]

where \( \mathbf{U} = \{\mathbf{U}_1, \mathbf{U}_2, \ldots, \mathbf{U}_N\} \) and \( \mathbf{V} = \{\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_m\} \). \( \mathbf{W}_{ij} \) controls the contribution from \( \mathbf{R}_{ij} \), and the term \( ||\mathbf{U}||_F + ||\mathbf{V}||_F \) is added to avoid overfitting.

#### 4.1 Capturing Local Information from Signed Social Networks
The local information in signed social networks is about the preference relations between users, and their “friends” (or users with positive links) and “foes” (or users with negative links). Next, we introduce our approach to capture local information from signed social networks based on the findings of the previous section.

Let \( P_i \) and \( N_i \) be user \( u_i \)’s friend circle, including users who have positive links with \( u_i \), and foe circle, including users who have negative links with \( u_i \), respectively. Based on \( P_i \) and \( N_i \), we can divide users into three groups as below:

- \( OP \) includes users who have only positive links as - \( OP = \{ u_i | P_i \neq \emptyset \land N_i = \emptyset \} \);
- \( ON \) includes users who have only negative links as - \( ON = \{ u_i | P_i = \emptyset \land N_i \neq \emptyset \} \);
- \( PN \) contains users who have both positive and negative links as - \( PN = \{ u_i | P_i \neq \emptyset \land N_i \neq \emptyset \} \).

We define \( \overline{U}_i^p \) and \( \overline{U}_i^n \) as the average user preferences of \( u_i \)’s friend circle and foe circle, respectively, as follows:

\[
\overline{U}_i^p = \frac{\sum_{u_j \in P_i} S_{ij} U_j}{\sum_{u_j \in P_i} S_{ij}}, \quad \overline{U}_i^n = \frac{\sum_{u_j \in N_i} S_{ij} U_j}{\sum_{u_j \in N_i} S_{ij}}
\]

(9)

where \( S_{ij} \) is the connection strength between \( u_i \) and \( u_j \).

- For a user \( u_i \) with only friend circle (or \( u_i \in OP \)), our previous finding suggests that \( u_i \)’s preference is likely to be similar with her friend circle. Hence, we force \( u_i \)’s preference close to \( P_i \) by minimizing the following term:

\[
\min \| U_i - \overline{U}_i^p \|^2.
\]

(10)

- For a user \( u_i \) with only foe circle (or \( u_i \in ON \)), this user is likely to be untrustworthy and we should not consider this user for the purpose of recommendation [37]. Therefore, we ignore local information from these users with only foe circles, which are only a small portion of the users in real-world signed social networks. For example, in the two studied datasets, there are less than 5% of users with only foe circles.

- For a user \( u_i \) with both friend and foe circles, our previous finding suggests that the preference of \( u_i \) is likely to be closer to that of his/her friend circle than that of his/her foe circle. In other words, (1) if a user \( u_i \) sits closer to his/her friend circle \( P_i \) than his foe circle \( N_i \), i.e., \( \| U_i - \overline{U}_i^p \|^2 - \| U_i - \overline{U}_i^n \|^2 < 0 \), we should not penalize this case while (2) if a user \( u_i \) sits closer to his/her foe circle \( N_i \) than her friend circle \( P_i \), i.e., \( \| U_i - \overline{U}_i^n \|^2 - \| U_i - \overline{U}_i^p \|^2 > 0 \), we should add a penalty to pull \( u_i \) closer to \( P_i \) than \( N_i \). Therefore, we propose the following minimization term to force \( u_i \)’s preference closer to \( P_i \) than \( N_i \) as:

\[
\min \max(0, \| U_i - \overline{U}_i^p \|^2 - \| U_i - \overline{U}_i^n \|^2)
\]

(11)

Next, we give details on the inner workings of Eq. (11). (1) When \( u_i \) sits closer to his/her friend circle \( P_i \) than his/her foe circle \( N_i \), the minimizing term in Eq. (11) is 0 because \( \| U_i - \overline{U}_i^p \|^2 - \| U_i - \overline{U}_i^n \|^2 < 0 \) and we do not add any penalty; and (2) when \( u_i \) sits closer to her foe circle \( N_i \) than her friend circle \( P_i \), the minimizing term in Eq. (11) is \( \| U_i - \overline{U}_i^p \|^2 - \| U_i - \overline{U}_i^n \|^2 > 0 \) because \( \| U_i - \overline{U}_i^p \|^2 - \| U_i - \overline{U}_i^n \|^2 > 0 \) and Eq. (11) will pull \( u_i \) back to \( P_i \) from \( N_i \).

We can develop a unified term to capture local information from these three groups in signed social networks with the following observations - (1) if we define \( \overline{U}_i^p = U_i \) for \( u_i \) in \( OP \), the term for \( OP \) is equivalent to \( \max(0, \| U_i - \overline{U}_i^n \|^2 - \| U_i - \overline{U}_i^n \|^2) \); and (2) if we define \( \overline{U}_i^p = U_i \) for \( u_i \) in \( ON \), the term \( \max(0, \| U_i - \overline{U}_i^n \|^2 - \| U_i - \overline{U}_i^n \|^2) \) is 0 for \( ON \), which indicates that we ignore the impact of users from \( ON \). Therefore by redefining \( \overline{U}_i^p \) and \( \overline{U}_i^n \) as,

\[
\overline{U}_i^p = \left\{ \begin{array}{ll}
\frac{\sum_{u_j \in P_i} S_{ij} U_j}{\sum_{u_j \in P_i} S_{ij}} & \text{for } u_i \in OP \cup PN, \\
U_i & \text{for } u_i \in ON.
\end{array} \right.
\]

\[
\overline{U}_i^n = \left\{ \begin{array}{ll}
\frac{\sum_{u_j \in N_i} S_{ij} U_j}{\sum_{u_j \in N_i} S_{ij}} & \text{for } u_i \in ON \cup PN, \\
U_i & \text{for } u_i \in OP,
\end{array} \right.
\]

(12)

we can find a unified term to capture local information from signed social networks as:

\[
\min \sum_{i=1}^{n} \max(0, \| U_i - \overline{U}_i^p \|^2 - \| U_i - \overline{U}_i^n \|^2)
\]

(13)

### 4.2 Capturing Global Information from Signed Social Networks

The global information of a signed social network reveals the reputation of a user in the whole network [20]. User reputation is a sort of status that gives additional powers and capabilities in recommender systems [31]. There are many algorithms to calculate the reputations of nodes in positive networks [24, 8]. However, a small number of negative links can significantly affect the status of the nodes, which suggests that we should consider negative links. Therefore, we choose a variant of PageRank, Exponential Ranking [36], taking into account negative links to calculate user reputations. In detail, we first perform Exponential Ranking to rank users by exploiting the global information of signed social networks. We assume that \( r_i \in \{1, 2, \ldots, N\} \) is the reputation ranking of \( u_i \) where \( r_i = 1 \) denotes that \( u_i \) has the highest reputation in the social network. Then we define user reputation score \( w_i \) as a function \( f \) of user reputation ranking \( r_i \): \( w_i = f(r_i) \) where the function \( f \) limits the value of the reputation score \( w_i \) within \([0, 1]\) and is a decreasing function of \( r_i \), i.e., top-ranked users have high reputation scores.

In the physical world, user reputation plays an important role in recommendation. Many companies employ people with high reputations to enhance consumers’ awareness and understanding of their products. Seno and Lukas found that suggestions from people with high reputations positively affect a consumer’s adoption of a brand [26]. While in the online world, Massa found that recommendations from users with high reputations are more likely to be trustworthy [20]. To capture global information from signed social networks, we can use user reputation scores to weight the importance of their recommendations. Originally the importance of \( R_{ij} \) in Eq. (8) is controlled by \( W_{ij} \). With signed social networks, we should also consider the reputation of \( u_i \); hence we define the new weight for \( R_{ij} \) as \( W_{ij} = g(W_{ij}, w_i) \) where \( g \) is a function to combine two weights. With these new weights,
the formulation to capture global information from signed social networks is computed as follows:

\[
\min \sum_{i=1}^{N} \sum_{j=1}^{m} g(W_{ij}, w_i) \|R_{ij} - U_i^T V_j\|^2_2 + \alpha(\|U\|_F^2 + \|V\|_F^2) \\
+ \beta \sum_{i=1}^{n} \max(0, \|U_i - \bar{U}_i\|^2_2 - \|U_i - \bar{U}_i\|^2_2)
\]  

(14)

where the importance of \(R_{ij}\) is controlled by \(W_{ij}\) and the reputation score of \(u_i\) through a function \(g\).

5. AN OPTIMIZATION ALGORITHM FOR RECSSN

We have introduced our approaches to capture local and global information from signed social networks. With these model components, we propose a recommendation framework, RecSSN, which exploits local and global information simultaneously from signed social networks. The proposed RecSSN framework solves the following optimization problem:

\[
\min \sum_{i=1}^{N} \sum_{j=1}^{m} g(W_{ij}, w_i) \|R_{ij} - U_i^T V_j\|^2_2 + \alpha(\|U\|_F^2 + \|V\|_F^2) \\
+ \beta \sum_{i=1}^{n} \max(0, \|U_i - \bar{U}_i\|^2_2 - \|U_i - \bar{U}_i\|^2_2)
\]  

(15)

where \(\beta \sum_{i=1}^{n} \max(0, \|U_i - \bar{U}_i\|^2_2 - \|U_i - \bar{U}_i\|^2_2)\) captures local information from signed social networks and the parameter \(\beta\) controls its contribution. The term \(g(W_{ij}, w_i)\) is introduced to capture global information from signed social networks.

By setting \(g(W_{ij}, w_i) = W_{ij}\) and ignoring all negative links, the proposed formulation for RecSSN in Eq. (15) can be written as follows:

\[
\min \sum_{i=1}^{N} \sum_{j=1}^{m} W_{ij} \|R_{ij} - U_i^T V_j\|^2_2 + \alpha(\|U\|_F^2 + \|V\|_F^2) \\
+ \beta \sum_{i=1}^{n} \|U_i - \bar{U}_i\|^2_2
\]  

(16)

Interestingly, this formulation is equivalent to one of the state-of-the-art recommender systems with positive networks SocialMF [6]. Therefore, RecSSN provides a unified recommendation framework with unsigned and signed social networks.

Eq. (15) is jointly convex with respect to \(U\) and \(V\) and there is no nice solution in closed form due to the use of the max function. A local minimum can be obtained through following gradient descent optimization method, which usually works well for recommender systems [11]. We define \(M_i^k\) at the \(k\)-th iteration for \(u_i\) as follows:

\[
M_i^k = \begin{cases} 
1 & \|U_i - \bar{U}_i\|^2_2 - \|U_i - \bar{U}_i\|^2_2 > 0 \\
0 & \text{otherwise}
\end{cases}
\]  

(17)

Then, we use \(J\) to denote the objective function of Eq. (15) in the \(k\)-th iteration as follows:

\[
J = \sum_{i=1}^{N} \sum_{j=1}^{m} g(W_{ij}, w_i) \|R_{ij} - U_i^T V_j\|^2_2 + \alpha(\|U\|_F^2 + \|V\|_F^2) \\
+ \beta \sum_{i=1}^{n} \max(0, \|U_i - \bar{U}_i\|^2_2 - \|U_i - \bar{U}_i\|^2_2)
\]  

(18)

The derivates of \(J\) with respect to \(U_i\) and \(V_j\) are as follows:

\[
\frac{\partial J}{\partial U_i} = -2 \sum_j g(W_{ij}, w_i) (R_{ij} - U_i^T V_j) V_j + 2\alpha U_i \\
+ 2\beta \sum_{j' \in N_i} \frac{1}{1 - \gamma_i - \gamma_j} \sum_{j' \in N_i} S_{ij} U_j + 2\alpha V_j
\]  

(19)

The detailed algorithm is shown in Algorithm 1. In Algorithm 1, \(\gamma_i\) and \(\gamma_j\) are learning steps, which are chosen to satisfy Goldstein Conditions [23]. Next, we briefly discuss the algorithm. In line 1, we initialize latent factors of users \(U\) and items \(V\) randomly. In each iteration, we calculate \(\bar{U}_i^k, \bar{U}_j^k\) and \(M_i^k\) for \(u_i\) from line 3 to line 6. From line 7 to line 9, we update \(U\) and \(V\) using aforementioned update rules. After learning the user preference matrix \(U\) and the item characteristic matrix \(V\) via Algorithm 1, an unknown score \(R_{ij'}\) from the user \(u_i\) to the item \(v_j'\) will be predicted as \(R_{ij'} = U_i^T v_j'\).

**Algorithm 1:** The Proposed Recommendation Framework RecSSN with Signed Social Networks.

**Input:** The rating information \(R\), positive links \(A_p\), negative links \(A_n\), the number of latent factors \(K\) and \(\beta\)

**Output:** The user preference matrix \(U\) and the item characteristic matrix \(V\)

1. Initialize \(U\) and \(V\) randomly and set \(k = 1\)
2. while Not converged do
3. for \(i = 1 : N\) do
4. Calculate \(\bar{U}_i^k\) and \(\bar{U}_j^k\) according to Eq. (12)
5. Calculate \(M_i^k\) according to Eq. (17)
6. end for
7. Calculate \(\frac{\partial J}{\partial U_i} \) and \(\frac{\partial J}{\partial V_j}\)
8. Update \(U \leftarrow U - \gamma_i \frac{\partial J}{\partial U_i}\)
9. Update \(V \leftarrow V - \gamma_j \frac{\partial J}{\partial V_j}\)
10. \(k = k + 1\)
11. end while
6. EXPERIMENTAL RESULTS

In this section, we conduct experiments to answer the following two questions - (1) can the proposed RecSSN framework improve the recommendation performance by exploiting signed social networks? and (2) which model components of RecSSN contribute to the performance improvement? Before answering these questions, we begin by introducing the experimental settings.

6.1 Experimental Settings

In Epinions, the scores in the user-item matrix denote the rating scores from users to items. Following common ways to assess recommendation performance in rating systems, we choose two metrics, corresponding to the Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE), which are formally defined as follows:

\[
RMSE = \sqrt{\frac{\sum_{(u_i,v_j) \in \mathcal{T}} (R_{ij} - \hat{R}_{ij})^2}{|\mathcal{T}|}},
\]
\[
MAE = \frac{1}{|\mathcal{T}|} \sum_{(u_i,v_j) \in \mathcal{T}} |R_{ij} - \hat{R}_{ij}|,
\]

where \( \mathcal{T} \) is the set of ratings in the testing set, \(|\mathcal{T}|\) is the size of \( \mathcal{T} \) and \( \hat{R}_{ij} \) is the predicted rating from \( u_i \) to \( v_j \). A smaller RMSE or MAE value means better performance. Note that previous work demonstrated that small improvement in RMSE or MAE terms can have a significant impact on the quality of the top few recommendations [9]. In this work, we choose \( x\% \) of rating scores as training and the remaining \( 1-x\% \) as testing, and \( x \) is varied as \{50, 70, 90\}.

In Slashdot, scores in the user-item matrix indicate whether users are associated with certain items. In this scenario, the performance is often evaluated via precision@N and recall@N [27], which are formally defined as follows:

\[
\text{precision@N} = \frac{\sum_{u \in \mathcal{U}} |\text{TopN}_u \cap I_i|}{\sum_{u \in \mathcal{U}} |\text{TopN}_u|},
\]
\[
\text{recall@N} = \frac{\sum_{u \in \mathcal{U}} |\text{TopN}_u \cap I_i|}{\sum_{u \in \mathcal{U}} |I_i|},
\]

where \( \text{TopN}_u \) is the set of \( N \) items recommended to user \( u \), that \( u \) has not been associated in the training set, and \( I_i \) is the set of items that have been associated with \( u \) in the testing set. A larger precision@N or recall@N value means better performance. The values of precision@N and recall@N are usually small in the case of sparse datasets. For example, the precision@5 is less than 0.05 over a dataset with 8.02e−3 density [40]. In this work, we set \( N = 5 \) and \( N = 10 \).

6.2 Performance Comparison of Recommender Systems

To answer the first question, we compare the proposed RecSSN framework with existing recommender systems. Traditional collaborative filtering systems can be grouped into memory-based systems and model-based systems; hence we choose two groups of baseline methods.

The first group of baseline methods includes the following memory-based systems:

- **UCF**: This system makes recommendations by aggregating recommendations from ones’ similar users only based on the user-item matrix.

- **pUCF**: This system is a variant of UCF, which combines recommendations from ones’ similar users and their friends [20]. pUCF utilizes both user-item matrix and positive links.

- **pnUCF**: This system is a variant of pUCF, which excludes recommendations from ones’ foes by exploiting negative links [37]. pnUCF makes use of user-item matrix, positive and negative links.

The second group of baseline methods includes the following model-based systems:

- **MF**: This system performs matrix factorization on the user-item matrix as shown in Eq. (8) [25]. It only utilizes the user-item matrix.

- **SocialMF**: This system combines both user-item matrix and positive links for recommendation [6], which is a special case of the proposed framework with only positive links as shown in Eq. (16).

- **SoReg**: This system also leverages both user-item matrix and positive links, and defines social regularization to capture positive links [17].

- **LOCABAL**: This system captures local and global information of positive links under the matrix factorization framework [31].

- **disSoReg**: In [15], two systems are proposed to exploit positive and negative links, respectively. disSoReg is a combination of these two systems to exploit positive and negative links simultaneously, which is actually a variant of SoReg by considering negative links as dissimilarity measurements.

Note that we use cross-validation to determine parameters for all baseline methods. For RecSSN, \( \beta \) is set to 0.7 and 0.3 for Epinions and Slashdot, respectively. More details about parameter selection for RecSSN will be discussed in the following subsections. We empirically set \( \alpha = 0.1 \) and the number of latent factors \( K = 10 \) for both datasets. In Eq. (14), we empirically find that \( f(x) = \frac{1}{\log(x+1)} \) and \( g(x,y) = x + y \) work well. The comparison results are demonstrated in Tables 4 and 5 for Epinions and Slashdot, respectively.

We make the following observations:

- In general, model-based methods outperform memory-based methods on the two studied datasets. Most of the existing recommender systems suffer from the data sparsity problem but model-based methods are usually less sensitive than memory-based methods [9].

- **pUCF** outperforms UCF. Furthermore, **SocialMF**, **SoReg** and **LOCABAL** outperform **MF**. These results support the known contention that exploiting positive links can significantly improve recommendation performance.

- **LOCABAL** exploits local and global information from positive links, and obtains better performance than the systems which model only local information from positive links such as **SocialMF** and **SoReg**. These observations indicate the importance of global information for recommendation.
Table 4: Comparison of Different Recommender Systems in Epinions

<table>
<thead>
<tr>
<th>Training</th>
<th>Metrics</th>
<th>Memory-based Methods</th>
<th>Model-based Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>MAE</td>
<td>1.0232</td>
<td>0.9764</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>1.2005</td>
<td>1.1477</td>
</tr>
<tr>
<td>70%</td>
<td>MAE</td>
<td>1.0074</td>
<td>0.9493</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>1.1758</td>
<td>1.1391</td>
</tr>
<tr>
<td>90%</td>
<td>MAE</td>
<td>0.9817</td>
<td>0.9272</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>1.1592</td>
<td>1.1059</td>
</tr>
</tbody>
</table>

Table 5: Comparison of Different Recommender Systems in Slashdot

<table>
<thead>
<tr>
<th>Metrics</th>
<th>Memory-based Methods</th>
<th>Model-based Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **pnUCF** obtains better performance than **pUCF**, which suggests that excluding recommendations from users with negative links can improve recommendation performance. Furthermore, **disSoReg** performs worse than **SoReg**. These results suggest that we may not consider negative links as dissimilarities in recommendation, which is consistent with observations in [33].

- The proposed RecSSN framework always obtains the best performance. RecSSN captures local and global information from signed social networks. In addition to positive links, signed social networks also provide negative links. More details about the effects of negative links on the performance of RecSSN will be discussed in the following subsection.

With these observations, we can draw conclusions about the first question - the proposed RecSSN framework outperforms the state-of-the-art recommender systems by exploiting local and global information from signed social networks.

### 6.3 Impact of Negative Links on RecSSN

We will now focus on the second issue of examining the precise impact of negative links on RecSSN. The experimental results in the previous subsection show that the proposed RecSSN framework outperforms various representative recommender systems with unsigned social networks. Compared to these systems, RecSSN also leverages information from negative links. In this subsection, we investigate the impact of negative links on the proposed RecSSN framework to answer the second question. In particular, we eliminate the effects of negative links systematically from RecSSN by defining the following algorithmic variants:

- **RecSSN\GN** - Eliminating the effect of negative links from global information of signed social networks by using Pagerank to calculate status scores of users with only positive links.

- **RecSSN\LN** - Eliminating the effect of negative links from local information of signed social networks by replacing \( \sum_{i=1}^{n} \max(0, \|u_i - \hat{u}_i\|_2^2 - \|u_i - \bar{u}_i\|_2^2) \) with \( \sum_{i=1}^{n} \|u_i - \hat{u}_i\|_2^2 \) in Eq. (15).

- **RecSSN\GN-LN** - Eliminating the effects of negative links from global and local information of signed social networks.

The parameters in all these variants are determined via cross-validation. The experimental results in Epinions are demonstrated in Figure 1. Note that we only show the results in Epinions because similar results were obtained in Slashdot. In general, eliminating any model component which captures the effect of negative links will reduce the recommendation performance. The relative performance reductions for variants compared to RecSSN are shown in Table 6. When eliminating the effect of global information of negative links from the proposed framework, the performance of **RecSSN\GN** degrades. We make a similar observation for **RecSSN\LN** when eliminating the effect of local information. For example, compared to RecSSN, **RecSSN\GN** and **RecSSN\LN** have 1.02% and 3.06% relative performance reductions, respectively, in terms of RMSE with 50% of Epinions data. When eliminating the effects of negative links from global and local information of signed social networks, **RecSSN\GN-LN** obtains worse performance than both **RecSSN\GN** and **RecSSN\LN**. This suggests that local and global information contain complementary information to each other for recommendation.

With the results from Figure 1 and Table 6, we can answer the second question - both local and global information of negative links in the proposed RecSSN framework can help improve the recommendation performance.

### 6.4 Parameter Analysis for RecSSN

The parameter \( \beta \) controls the contribution of local information in signed social networks. In this subsection, we investigate how changes of \( \beta \) affect the performance of RecSSN. We vary the value of \( \beta \) as \( \{0, 0.001, 0.01, 0.1, 0.3, 0.5, 0.7, 1, 10\} \). The results in Epinions w.r.t. RMSE and MAE are demonstrated in Figures 2(a) and 2(b), respectively. Since we have similar observations in Slashdot, we only show the results in Epinions to save space.

With increase in \( \beta \), the importance of local information is increased. We make the following observations:

- **RecSSN\LN** - Eliminating the effect of negative links from local information of signed social networks.
The performance first increases rapidly, which suggests that local information is helpful in improving recommendation performance in signed social networks.

- When $\beta$ varies from 0.3 to 0.7, the performance is relatively stable. This property is useful from a practical point of view because it makes it easier to set $\beta$.

- After this point, the performance reduces. When $\beta$ increases from 1 to 10, the performance reduces dramatically. A large value of $\beta$ will lead to local information dominating the learning process. In such cases, the estimates of the user preference matrix $U$ and the item characteristic matrix $V$ will overtime to the local information in signed social networks. For example, when $\beta \to \infty$, the user preference matrix $U$ is learned only from signed social networks and the item characteristic matrix $V = 0$.

7. RELATED WORK

The pervasive nature of social media provides independent sources of information, which brings new opportunities for recommendation. Recently, social relations have found increasing importance from the perspective of improving recommendation performance [20, 16, 17, 7]. In [16], a matrix-factorization system, referred to as SoRec, is proposed. It performs a co-factorization on the user-item ratings matrix and user-user social relation matrix by assuming that users should share the same user preference vectors in the rating space and the social relation space. Trust Ensemble is introduced in [18] to take advantage of strong dependency connections. It assumes that a user’s online behavior can be affected by his/her trusted friends on the Web, and, based on this intuition, unknown ratings for a certain user are predicted by the user’s characteristics and the user’s trusted friends’ recommendation. In [6], a social recommender system with trust propagation is proposed to recommend items for users in social network. The underlying assumption of this method is that directly connected users may have similar interests and thus it forces a user’s preference close to the average user preference of his/her social network. Social regularization is employed by [17] to exploit strong dependency connections for recommendation. This approach forces a user’s preference close to user preferences of his/her social networks. The low cost of social relation formation can lead to social relations with heterogeneous strengths [38]. Since users with strong strength are more likely to share similar tastes than those with weak strength, treating all social relations equally is likely to lead to degradation in recommendation performance. Therefore the closeness between a user’s preference and the preferences of his/her social network is controlled by their rating similarities [17]. These social recommender systems can reduce the number of cold-start users and improve recommendation performance [6].

8. CONCLUSIONS

The pervasively available social networks in social media have encouraged a large body of literature about recommendation. The vast majority of these recommender systems focus on unsigned social networks (or social networks with only positive links). However, social networks in social media could contain positive and negative links and little work exists for recommendation with signed social networks. The leveraging of negative links for recommendation is a challenging task because straightforward extensions of unsigned networks do not seem to be applicable in this case. In this paper, we first perform data-driven analysis on signed social networks and make a number of observations. Then we provide principled approaches to capture local and global information from signed social networks mathematically, which results in a novel recommendation framework, which we refer to as RecSSN. Experimental results demonstrate that the proposed framework outperforms various state-of-the-

![Figure 1: Impact of Negative Links on The Proposed Framework RecSSN in Epinions.](image)

Table 6: Relative Performance Reductions for Variants Compared to RecSSN.

<table>
<thead>
<tr>
<th>Variants</th>
<th>50% MAE</th>
<th>50% RMSE</th>
<th>70% MAE</th>
<th>70% RMSE</th>
<th>90% MAE</th>
<th>90% RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RecSSN\LN</td>
<td>-2.06%</td>
<td>-3.06%</td>
<td>-3.15%</td>
<td>-3.71%</td>
<td>-1.67%</td>
<td>-3.21%</td>
</tr>
<tr>
<td>RecSSN\GN</td>
<td>-2.59%</td>
<td>-3.29%</td>
<td>-3.56%</td>
<td>-3.22%</td>
<td>-2.04%</td>
<td>-3.56%</td>
</tr>
<tr>
<td>RecSSN\GN-LN</td>
<td>-0.88%</td>
<td>-1.02%</td>
<td>-0.98%</td>
<td>-1.21%</td>
<td>-0.92%</td>
<td>-1.15%</td>
</tr>
</tbody>
</table>
art recommender systems. Further experiments are conducted to understand the importance of signed social networks in the proposed RecSSN framework.

There are several directions, which might be investigated. First, the proposed RecSSN framework chooses matrix factorization as the basic model on top of which the algorithms are constructed. While this is a natural choice because of the well-known robustness of such systems, it would be instructive to investigate whether other types of models can be used. Second, as user preferences and signed social networks might evolve, incorporating temporal information into the proposed RecSSN framework is an interesting direction. Third, we make several important observations about signed social networks in this paper, which may be helpful in developing algorithms for other online applications of signed social networks, such as information propagation and spammer detection. Finally a comprehensive overview about signed network mining in [30] suggests that mining signed networks is still in its early stage; thus we would like to investigate more applications in signed networks.

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9. REFERENCES


